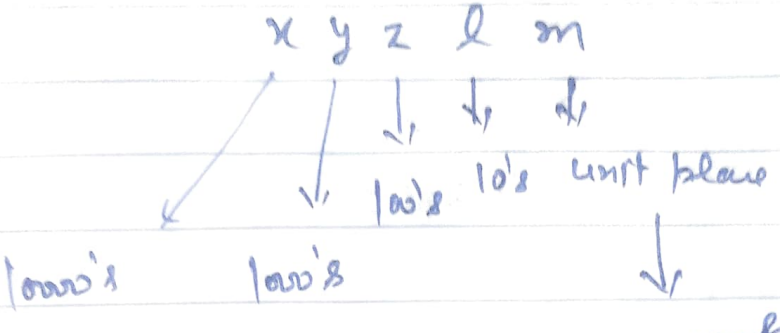
* Unit digit *

The means of the unit digit is to find the unit place of a digits lets say for example.



Now first we will find the cycle of the number.

1. $0^n \rightarrow 0$

2. $1^n \rightarrow 1$

3. $2^1 \rightarrow 2$
   $2^2 \rightarrow 4$
   $2^3 \rightarrow 8$
   $2^4 \rightarrow 16$
   $2^5 \rightarrow 32$

So cycle of 2 is 4.

Cycle of the number as follows :→

$2 \rightarrow 4$	$7 \rightarrow 4$
$3 \rightarrow 4$	$8 \rightarrow 4$
$4 \rightarrow 2$	$9 \rightarrow 2$
$5 \rightarrow 1$	$6 \rightarrow 1$

Some Questions on unit digit :→

Q :→ $123 + 345 + 780 + 65 + 44$

$3 + 5 + 0 + 5 + 4 \rightarrow 17$

7 is the answer.

Q :→ $676 \times 543 \times 19 \rightarrow 6 \times 3 \times 9$

$\rightarrow 162$

2 is the answer.

Q :→ 2^{35}

cycle of 2 is 4. then $4)35(8$
32
3

$2^3 \rightarrow 8$ Ans

Q $2^{36} \rightarrow 4)36(9$
36
0

$\rightarrow 2^4 \rightarrow 16$ Ans

Q :→ $(12)^{78} \rightarrow 2^{78} \rightarrow 4)78(19$
4
38
36
2

$2^2 \rightarrow 4$ Ans

Q:- $5^{47} + 7^{52}$

$4)\overline{47}\,(11$ — 4, 7, 4, 3

$4)\overline{52}\,(13$ — 4, 12, 12, 0

$3^3 \rightarrow 27$

$7^4 \rightarrow 7 \times 7 \times 7 \times 7$ → 49, 49 → 01

$27 + 01 = 28$ → 8

Ans

Q:- Find the unit digit of 111!

Sol:- $1 \times 2 \times 3 \times 4 \times 5 \times \ldots\ldots$ (2 × 5 = 10)

that means last digit will be zero

Note: 5! to ---- all value with ! last digit will zero.

Q:- Find the unit digit of the product of all prime number b/w 1 and (11)"

Prime no → (1, 2, 3, 5, 11, 13 ---- 11")

2 × 5 = 10

So, It will give the 0 as the unit digit.

Q1:→ Find last digit of $222^{888} + 888^{222}$

Sol^{n}

222^{888}

$\Rightarrow 2^{888}$ → 4) 888 (222
8
8
8
8
8
0

$\Rightarrow 2^4 \rightarrow 16$

8^{222} → 4) 222 (55
20
22
20
2

$8^2 \rightarrow 64$

$6+4 \rightarrow 10$

Q2:→ Find unit digit of

$1^2 + 2^2 + 3^2 + \cdots 100^2$

1 11
2 1
3 1
1
1
1
10 1 20 30 — — —

$1^2 + 2^2 + 3^2 + 4^2 + \cdots 10^2$

$\Rightarrow 1+4+9+16+25+36+49+64+81+100$

$\Rightarrow \underline{45}$

Last digit is 5 that will similar to 10 column.

Then $5 \times 10 \rightarrow 5\underline{0} \rightarrow \boxed{0}$ is the unit digit.

Note

1	11			91
2	12			
1	1			
1	1			
1	1			
10	20			100

Q! → $1^1 + 2^2 + 3^3 + 4^4 + \text{——} 10^{10}$ find the unit digit.

$1^1 \rightarrow 1$ | $6^6 \rightarrow 6$

$2^2 \rightarrow 4$ | $7^7 \rightarrow 3$

$3^3 \rightarrow 7$ | $8^8 \rightarrow 6$

$4^4 \rightarrow 6$ | $9^9 \rightarrow 9$

$5^5 \rightarrow 5$ | $10^{10} \rightarrow 0$

$1+4+7+6+5+6+3+6+9+0$

$\Rightarrow 47 \rightarrow (7)$ is the answer.

Q! → $\frac{10!}{100}$ find unit digit

$$\frac{10!}{100} = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{2^2 \times 5^2}$$

$\Rightarrow 1 \times 2^6 \times 3^4 \times 7 \Rightarrow$

$1 \times 4 \times 1 \times 7 \rightarrow 2(8) \rightarrow (8)$ is the answer.

Q:- what is the unit digit of the sum of first 111 whole number.

Sol^{n} $0 + 1 + 2 + 3 + \cdots \underline{110}$

$\frac{n(n+1)}{2} \rightarrow \frac{110 \times 111}{2} \rightarrow 55 \times 111$

$\rightarrow$ (5) is the answer.

Q:- find the unit digit of sum of first 111 natural number.

Sol^{n} $1 + 2 + 3 + \cdots \underline{111}$.

$\frac{n(n+1)}{2} \rightarrow \frac{111 * 112}{2} \rightarrow 111 \times 56 \rightarrow$ (6) is the answer

Q:- find the unit digit in $1! + 2! + 3! + 4! + \cdots 50!$

Sol^{n} $\rightarrow$ $1! + 2! + 3! + 4! + 5! + \cdots 50!$

$1! \rightarrow 1$

$2! \rightarrow 2$

$3! \rightarrow 6$

$4! \rightarrow 24$

$5! \rightarrow 120$

$50! \rightarrow 0$

$1 + 2 + 6 + 4$

$\rightarrow$ 1(3) is the answer.

Note: → for find the no. of Perfect Square b/w two number find out near by Perfect Sq. search out then subtract them.

Q1 → How many Perfect Sqr. having digit 6 at the unit place between 1 to 600.

Sol^{n} [1 to 600]

- 4 → 16 — 24 → 576
- 6 → 36
- 14 → 196 — So, there are total 5 perfect Sqr.
- 16 → 256

Q2 → if $a + 1/a$ = 14.28% of 42, then find the unit digit of the expression $a^2 + 1/a^2$

Sol^{n} $a + \frac{1}{a} \Rightarrow \left(a + \frac{1}{a}\right)^2 \rightarrow a^2 + \frac{1}{a^2} + 2 \cdot a \cdot \frac{1}{a}$

$$a^2 + \frac{1}{a^2} + 2 = K \quad \Big| \quad a^2 + \frac{1}{a^2} = K - 2 \text{ ---- i)}$$

$a + \frac{1}{a}$ = 14.28 % 42 $\Rightarrow \frac{1}{7} \times 42$ (6)

$a + \frac{1}{a} = (6) \rightarrow (K)$ $\quad [6^2 = 36]$

$a^2 + \frac{1}{a^2} = 36 - 2 = 3(4)$

4 is the answer.